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# THE UNIVERSITY OF MICHIGAN

## COLLEGE OF ENGINEERING

DEPARTMENT OF AERONAUTICAL AND ASTRONAUTICAL ENGINEERING  
AIRCRAFT PROPULSION LABORATORY

*Quarterly Progress Report No. 1  
(1 June 1962 to 31 August 1962)*

### *The Feasibility of Rotating Detonation Wave Rocket Motor*

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*Under contract with:*

Air Force Flight Test Center  
6593d Test Group Development  
Contract No. AF 04(611)-8503  
Edwards Air Force Base, California

*Administered through:*

*August 1962*

OFFICE OF RESEARCH ADMINISTRATION • ANN ARBOR

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COLLEGE OF ENGINEERING  
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## NOMENCLATURE

### PART II-A

$f_j$	spray distribution function type $j$
$\underline{x}$	position vector in phase space
$r$	radius of a droplet
$\underline{V}$	velocity vector of a droplet
$t$	time
$\rho$	density
$We$	Webber number
$\underline{u}$	velocity of the gas
$S$	surface tension
$p$	pressure
$h$	enthalpy per unit mass
$W$	molecular weight
$Y$	mass fraction of a component of the gas
$T$	temperature
$M$	Mach number
$Z$	mass flux fraction of spray

### Subscripts

$f$	gaseous fluid
$g$	gas
$j$	$j^{\text{th}}$ type of droplet
$cr$	critical



## NOMENCLATURE (Continued)

### PART II-A (Concluded)

$l$	liquid
$o$	upstream side of wave where conditions are spatially uniform
$\infty$	downstream side of wave where conditions are spatially uniform
$k$	a chemical species

### PART II-B

$\rho_1$	densities of unburned gas
$V_c$	velocity of burned gas relative to detonation
$V_D$	detonation velocity
$h$	enthalpy per unit mass
$p$	pressure
$\mathcal{R}$	universal gas constant
$\gamma$	molecular weight
$M$	Mach number
$T$	temperature
$q_w$	heat flux per unit area at wall
$\tau_w$	shear stress at the wall
$\mu$	viscosity
$\nu$	kinematic viscosity
$Q_w$	total heat flux during heat pulse
$\bar{q}_w$	average heat flux per unit area during pulse
$c$	ratio of heat pulse width to wavelength

## NOMENCLATURE (Concluded)

### PART II-B (Concluded)

$\bar{Q}_w$	overall average heat flux per unit area to walls of chamber
$O( )$	means order of
$\alpha$	thermal diffusivity $k/\rho C_s$
$k$	thermal conductivity
$C_s$	specific heat of wall material
$\ell$	thickness of combustion chamber walls
$e$	conditions downstream of the detonation
$l$	conditions upstream of the detonation

## FOREWORD

This report represents the first quarterly progress report, 1 June 1962 to 31 August 1962, on Contract AF 04(611)-8503, a contract between Edwards Air Force Base and The University of Michigan. The aim of this contract is to investigate the feasibility of a rotating detonation wave rocket motor. This project is directed by Professors J. A. Nicholls and R. E. Cullen of The University of Michigan. The Air Force Project Engineer is Richard Weiss (DGRR), 6593d Test Group (Development), Edwards, California.

## SUMMARY

This report presents the work accomplished on the rotating detonation wave rocket engine feasibility program during the period 1 June 1962 to 31 August 1962.

The work described in this report represents the beginning of the experimental and theoretical studies that will consume approximately the first half of the program. The last half of the program will be concerned with the design of a nominal 1000 lb thrust hydrogen-oxygen rocket motor utilizing the detonative mode of combustion.

The theoretical studies have included two significant problems:

(1) A study of the structure of a detonation wave through a two phase medium; fuel droplets in an oxidizer atmosphere or oxidizer droplets in a fuel atmosphere, has yielded some tentative conclusions. The theoretical analysis of Williams<sup>10</sup> considers only the heterogeneous reaction (i.e., the gaseous phase is exclusively composed of either fuel or oxidizer), and does not account for the possibility of droplet shattering. The relatively leisurely rates of evaporation led him to conclude that a stable detonation wave in a heterogeneous medium was doubtful, unless the droplet size was very small. However, some very limited experimental results would seem to indicate that droplet shattering could be influential in furnishing sufficiently small droplets to allow for the attainment of a stable wave.

(2) A study of the heat transfer to the wall of a rotating detonation wave combustion chamber, for the simplified model considered, has resulted in the preliminary conclusion that it is of the same order as that encountered in the throat section of a conventional rocket engine. Also, it appears that the fluctuation of the wall temperature will be negligible for the case considered.

The following experimental studies are, for the most part, in the instrumentation and setup phase. The areas under study are:

(1) A study of the pressure and temperature effects on the detonation wave properties of a hydrogen-oxygen mixture is being carried out because the detonation properties, e.g., Mach number of propagation and the pressure ratio across the wave, are strong functions of the initial temperature of the mixture. The low temperatures and high pressures will also make it possible to study the effects of an imperfect gas on the detonation characteristics of the mixture.

A description of the experimental apparatus is presented.

(2) Inasmuch as a small experimental rotating detonation wave rocket motor

was available,\* it was decided to use it in further tests in order to support the theoretical and experimental studies presently under way. Also, these tests will serve as a means of checking instrumentation and techniques to be employed in the final testing phase of the 1000 lb thrust motor.

(3) In order to intelligently design a final test model it was deemed necessary to consider the effect of geometry on the propagation characteristics of a detonation wave. The geometrical tests will consider the effect of curvature (pressure gradient) and the effect of various degrees of one dimensional relief.

(4) A study of detonation waves in heterogeneous media is being initiated. The experiment is considered to be quite important because of the question of stable propagation. The detonation characteristics, e.g., Mach number of detonation and the pressure and temperature rise across the detonation front, are expected to change. These detonation properties must be well defined before a valid design of the experimental rotating detonation wave rocket motor can be finalized.

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\*This motor has been run on many previous occasions and therefore, should prove to be a valuable vehicle to carry out these objectives.

## I. INTRODUCTION

The normal deflagration of combustible gaseous mixtures takes place at a leisurely rate that is governed by the multiple diffusion processes within and in the neighborhood of the combustion front. These rates, or flame speeds, are in the order of a few feet per second. Under favorable conditions, combustion waves can be developed that possess extraordinary velocities of thousands of feet per second. These combustion waves can be described as a shock wave followed by combustion; the shock wave produces temperatures that are sufficiently high to cause rapid ignition and burning of the mixture. Detonation waves can be produced in many ways, the most convenient being a sufficiently long tube into which a combustible mixture is introduced and ignited.

Once developed, usually in a short distance, the detonation wave propagates at a very regular velocity. This velocity is dependent upon the temperature, pressure, gaseous medium, and slightly on the size of the detonation tube. These waves are very amenable to analysis and the accuracy with which theoretical computations predict experiment results is surprising.

The continued use of the deflagration, rather than the detonation process, in rocket engine design, appears to be due more to convention and habit, than it is to any clear technical advantages. In fact, it appears that no technical comparison of the relative advantages of detonation and deflagration processes, as applied to rocket motor design, has yet been made. In light of the high total temperatures, and large heat releases per unit volume per unit time obtainable with detonation waves, it appears that a detonation process possesses some inherent advantages which should be investigated. Also, it seems possible that scaling techniques should be very simple with a detonation process because of the predictable behavior of detonation waves propagating into a mixture sufficiently removed from the detonability limits.

The earliest studies proposing the use of detonation waves in a propulsion device was in 1941 by Hoffman.<sup>1</sup> Recently Nicholls and co-workers<sup>2,3,4,5,6,7</sup> have studied intermittent gaseous detonation waves as a thrust producing mechanism. Also the applicability of the detonative combustion processes in ram-jet engines has been studied by Dunlap, Brehm and Nicholls,<sup>35</sup> and by Sargent and Gross.<sup>34</sup>

Recent experiments of partially unconfined detonation waves by Sommers,<sup>36</sup> and the recent success of Nicholls,<sup>8</sup> and Rhodes,<sup>9</sup> in stabilizing gaseous detonation waves has greatly improved the feasibility of using detonation waves in ram-jet and rocket motors.

To our knowledge, the first study made of the use of a detonative combustion process in rocket engines was performed by Morrison and Cosens<sup>37</sup> under the support of the Institute of Science and Technology of The University of Michigan.

Although this is the first application of detonative combustion to a rocket motor, Vorlasekhovskii<sup>38</sup> has studied multiple detonation waves in a circular channel.

Since the propellants are injected into the rocket combustion chamber in both a gaseous and liquid state, it appears that a two phase (liquid-gas) detonation process will occur rather than gaseous detonation. Therefore, studies concerning the existence, structure, and stability of detonation waves in sprays are critical to the study of the feasibility of a rotating detonation wave rocket motor. It will also be necessary to study the effect of geometry, curvature, and relief on the propagation characteristics of a detonation wave.

The heat transfer to the wall from a detonative combustion process is thought to be increased by a substantial factor over the heat transfer in deflagrative combustion systems. Therefore, the solution of the heat transfer problem seems to be of paramount importance to the successful design and feasibility testing of the nominal 1000 lb thrust rotating detonation wave rocket motor.

## II. THEORETICAL STUDIES

### A. DETONATION WAVE IN HETEROGENEOUS, LIQUID-GAS MEDIA

To date only a few studies of detonation waves in sprays have been made. Williams<sup>15</sup> has thoroughly investigated detonation waves in dilute sprays by applying a statistical method to spray combustion.<sup>10</sup> Webber<sup>11</sup> has found, through experimentation, that most of the cases of spray combustion in a shock tube lead to a high amplitude wave. Also, Cramer's<sup>12</sup> experiments have shown that droplet shattering can enable a detonation wave to propagate through a heterogeneous medium.

A brief review of these studies shall be presented in the following discussion.

In a dilute spray\* with  $We \ll We)_{cr}$ ,\*\* F. A. Williams has employed the spray distribution function,<sup>10,13</sup>

$$f_j(r, \underline{x}, \underline{V}, t) dr d\underline{x} d\underline{V} \quad (1)$$

which gives the probable number of droplets of composition  $j$  at time  $t$ , with a radius between  $r$  and  $r + dr$ , located in the position range  $d\underline{x} \equiv dx_1 dx_2 dx_3$  about  $\underline{x}$ , and with a velocity in the range  $d\underline{V} \equiv dV_1 dV_2 dV_3$  about  $\underline{V}$ .

\*The mass of fluid per unit spatial volume,  $\rho_f$ , is related to the actual mass of fluid per unit volume of space available to the gas,  $\rho_g$ , through the equation

$$\frac{\rho_f}{\rho_g} = 1 - \sum_{j=1}^M \int \int \frac{4}{3} \pi r^3 f_j dr d\underline{V}$$

The last term of the equation is the fraction of the total spatial volume occupied by the droplets. A dilute spray implies  $\rho_f \approx \rho_g$ .

\*\*The Webber number is defined as

$$We \equiv \frac{2r\rho_g |\underline{V} - \underline{U}|^2}{S}$$

where  $\underline{U}$  is the gas velocity and  $S$  is the surface tension. Hinze<sup>14</sup> found that if the Webber number exceeds some critical value,  $We)_{cr} \approx 20$ , the aerodynamic forces will cause the droplet to break up. Also, it was found that for  $We \ll We)_{cr}$  the droplets are nearly spherical.<sup>10</sup>



The change of  $f_j$  with time can be represented by

$$\frac{\partial f_j}{\partial t} = - \frac{\partial}{\partial r} (R_j f_j) - \nabla_{\mathbf{x}} \cdot (\underline{\mathbf{v}} f_j) - \nabla_{\mathbf{v}} \cdot (\underline{\mathbf{F}}_j f_j) + Q_j + \Gamma_j \quad (2)$$

where  $Q_j$  is a droplet source term,  $\Gamma_j$  is a droplet collision term,

$$\underline{\mathbf{F}}_j \equiv \left. \frac{d\underline{\mathbf{v}}}{dt} \right|_j \text{ and } R_j \equiv \left. \frac{dr}{dt} \right|_j$$

are the total forces on the droplet per unit droplet mass and the rate of droplet growth respectively. It is apparent that Eq. (2) must play the same role in spray combustion as the Boltzmann equation does in the mathematical theory of non-uniform gases, and that the  $f_j(r, \underline{\mathbf{x}}, \underline{\mathbf{v}}, t)$  is a function similar to the molecular distribution function.

In accordance with the assumption of dilute sprays, the equations governing the propagation of a detonation wave through a spray are the ordinary fluid dynamic equations with suitable modifications to account for the average effect of the droplets.<sup>10</sup> The spray distribution function is coupled with the equations of motion for the gaseous medium by the total force on the droplet per unit droplet mass and the rate of droplet growth. Then the general equations relating the change of fluid properties across a detonation wave, assuming uniform conditions at stations 0 and  $\infty$ , are written as follows:

$$\begin{aligned} \rho_{f\infty} U_{\infty} + \sum_{j=1}^M \iint \rho_{\ell, j\infty} \frac{4}{3} \pi r^3 V f_{j\infty} dr dV &= \rho_{f0} U_0 \\ &+ \sum_{j=1}^M \iint \rho_{\ell, j0} \frac{4}{3} \pi r^3 V f_{j0} dr dV \end{aligned} \quad (3)$$

$$\begin{aligned} \rho_{f\infty} U_{\infty}^2 + \sum_{j=1}^M \iint \rho_{\ell, j\infty} \frac{4}{3} \pi r^3 V^2 f_{j\infty} dr dV + p_{\infty} \\ = \rho_{f0} U_0^2 + \sum_{j=1}^M \iint \rho_{\ell, j0} \frac{4}{3} \pi r^3 V^2 f_{j0} dr dV + p_0 \end{aligned} \quad (4)$$

$$\begin{aligned}
& \rho_{f\infty} U_{\infty} (h_{f\infty} + \frac{U_{\infty}^2}{2}) + \sum_{j=1}^M \iint \rho_{\ell,j\infty} \frac{4}{3} \pi r^3 v (h_{\ell,j\infty} + \frac{v^2}{2}) f_{j\infty} dV dr \\
& = \rho_{fo} U_o (h_{fo} + \frac{U_o^2}{2}) + \sum_{j=1}^M \iint \rho_{\ell,j0} \frac{4}{3} \pi r^3 v (h_{\ell,j0} + \frac{v^2}{2}) f_{j0} dr dv
\end{aligned} \tag{5}$$

$$\frac{p_{\infty}}{\rho_{f\infty} T_{f\infty} \sum_{k=1}^N \frac{Y_{k\infty}}{W_k}} = \frac{p_o}{\rho_{fo} T_{fo} \sum_{k=1}^N \frac{Y_{ko}}{W_k}} \tag{6}$$

In order to derive the Rankine-Hugoniot equations for dilute sprays, Williams<sup>15</sup> proposed the following assumptions: (a) There are no droplet sources or sinks and no collisions between drops in the control volume being considered, (b) All droplets will disappear downstream of the detonation wave,  $f_{j\infty} = 0$ , (c) Initially all the droplets have the same velocity as the gas,  $f_{j0} \propto \delta(V - U_o)$ ,\* (d) The initial enthalpy per unit mass,  $h_{\ell,j}$ , of the droplets is independent of the drop diameter, (e) The initial and final average molecular weights of the gas are equal,

$$\sum_{k=1}^N \frac{Y_{k\infty}}{W_k} = \sum_{k=1}^N \frac{Y_{ko}}{W_k} ,$$

(f) The specific heats of all gaseous species are constant and equal over a temperature range including  $T_{fo}$ ,  $T_{f\infty}$  and a standard reference temperature  $T^\circ$ . Williams derived the Rankine-Hugoniot equations for dilute sprays by using the above assumptions and Eqs. (2) to (6).

Comparison of Williams' results with classical theory, Table I, shows that the Rankine-Hugoniot equations for detonations in dilute sprays differ from those in gaseous media only in the effective heat of reaction and in the equation of state.

The Chapman-Jouguet case for dilute sprays and gaseous media having the same amount of actual heat release and the same initial conditions were found, by Williams, to differ on the following points: (a) a Chapman-Jouguet detonation wave in a dilute spray propagates at a slightly higher Mach number, (b) the downstream pressure is slightly higher, about 10% for the case of the dilute spray,

\* $\delta$   $\equiv$  Dirac delta functions

The following table compares the Rankine-Hugoniot equations for heterogeneous and homogeneous media.

TABLE I  
RANKINE-HUGONIOT EQUATIONS FOR DILUTE SPRAYS AND GASEOUS MIXTURES

Williams' theory for dilute sprays	Classical theory for gaseous mixtures
$\rho_{\infty} U_{\infty} = \rho_0 U_0$	$\rho_{\infty}' U_{\infty}' = \rho_0 U_0'$
$\rho_{\infty} U_{\infty}^2 + p_{\infty} = \rho_0 U_0^2 + p_0$	$\rho_{\infty}' U_{\infty}'^2 + p_{\infty}' = \rho_0' U_0'^2 + p_0'$
$C_p T_{f\infty} + \frac{U_{\infty}^2}{2} = C_p (1 - Z_0) T_{f0} + \frac{U_0^2}{2} + Q$	$C_p' T_{\infty}' + \frac{U_{\infty}'^2}{2} = C_p' T_0' + \frac{U_0'^2}{2}$
$\frac{p_{\infty}}{\rho_{\infty} T_{\infty}} = \frac{p_0}{\rho_0 (1 - Z_0) T_{f0}}$	$\frac{p_{\infty}'}{\rho_{\infty}' T_{\infty}'} = \frac{p_0'}{\rho_0' T_0'}$

where

$$Q \equiv Z_0 C_p T_{f0} + \sum_{j=1}^M Z_{j0} h_{l,j0} + \sum_{k=1}^M \left[ \left\{ h_k^0 + C_p (T_{f0} - T^0) \right\} \left\{ Y_{k0} (1 - Z_0) - Y_{k\infty} \right\} \right]$$

or

$$Q \equiv Z_0 C_p T_{f0} + \hat{Q}$$

where  $\hat{Q}$  is the total heat released per unit mass of mixture and  $Z_0$  is the mass flux fraction of spray in front of the detonation wave

and (c) the dilute spray case has a slightly higher downstream temperature. Since the above comparison is based on constant actual total heat release, the differences must be the result of change in the equation of state and in the effective heat of reaction caused by the presence of the droplets. If a comparison is made between two systems, dilute spray and gaseous media, having the same initial temperature and comprised of the same fuel and oxidizer, the decrease in the total heat release caused by the latent heat of vaporization of the droplet will tend to erase the above differences.

In analyzing the steady, one-dimensional spray detonation structure,<sup>16</sup> additional assumptions are made for simplification: (a) all droplets are the same size and travel with the same velocity, i.e.,  $f(x, r, v) = n(x)\delta(r - \bar{r})\delta(v - \bar{v})$ , (b) the system is composed of non-volatile sprays in a gaseous atmosphere so that the burning processes are completed in the surface layer of the droplet, i.e., homogeneous reactions are negligible in comparison to the heterogeneous processes, (c) diffusion in the gas can be neglected, (d) the overall stoichiometry of the reactions occurring in the immediate neighborhood of each droplet does not change, and (e) the radial mass flux fraction at the outer edge of the surface layer of a droplet is independent of  $x$ . Using these additional assumptions Williams<sup>16</sup> found that the von Neumann detonation structure for gaseous detonations\* would be a valid approximation to the structure of a heterogeneous detonation wave. This conclusion was obtained by examining the characteristic length for such properties as gas temperature, gas velocity, droplet velocity, and the mass flux fraction of sprays. The order of magnitude of these characteristic lengths, for a spray composed of 30 $\mu$  fuel droplets in air is 10<sup>-4</sup>cm, 10<sup>-4</sup>cm, 10<sup>-4</sup>cm and 10<sup>-2</sup>cm respectively.

Due to the thickness of the reaction zone in a pure heterogeneous detonation process,<sup>16</sup> the interaction of the deflagration zone with the walls seems stronger than its interaction with the shock wave. Therefore, it is questionable whether the heterogeneous combustion could release sufficient heat to afford the wall losses and to support the shock front. Hence the stability of a spray-detonation must involve both the heterogeneous and homogeneous types of combustion.<sup>16</sup>

Due to the various assumptions in Williams theory, its application might be limited. In order to ascertain the extent of applicability, Williams theory will be compared with some experimental results.

Investigating heterogeneous combustion at high Reynolds numbers in a shock tube, Webber<sup>11</sup> found that an overly fine spray or a volatile fuel could produce a spontaneous explosion. Sprays of fairly coarse droplets or of relatively non-volatile liquid fuels might burn rapidly enough to sustain and amplify a pressure wave. These experiments show that at high Reynolds numbers the specific combustion rates depend on the rate at which atomized particles are torn off the droplet, and this specific combustion rate influences the stability of the pressure wave.

Cramer<sup>12</sup> continued Webber's work by studying the onset of detonations in a two-phase medium. The gross pattern of the structure of spray-detonation waves superficially appears similar to that found by Williams; it is approximately like the von Neumann detonation structure for a gaseous mixture. However, the detailed mechanism showed that shattering of large drops into small droplets by the high velocity gas behind the shock wave had a great influence on the stability of the

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\*A gaseous shock wave followed by a heterogeneous deflagration wave.

detonation wave. Williams' analysis of spray combustion does not consider this shattering phenomenon because it was assumed that the droplets had the property  $We \ll We_{cr}$ . This is one reason why Williams found the stabilization of spray detonation waves doubtful.\*

It is possible to generalize Williams' theory to include the case when droplet breakup phenomenon is prevailing. The necessary assumptions are: (a) the droplets retain their spherical shape until they arrive at a critical flow condition at which time they instantly breakup, (b) a source term,  $Q$ , describing this instantaneous disappearance and creation of particles be introduced into the governing equations.\*\*

The sequence of events observed by Cramer\*\*\* are as follows: (a) the high velocity gas behind the shock wave has a displacement as well as a shattering effect on the droplets, (b) the heterogeneous non-uniform spray is subjected to a transient redistribution behind the shock wave because of the displacement phenomenon, (c) the redistribution of droplets causes the combustion process to pass through three zones. These zones are shown in Fig. 1. The first zone has a high number density while the second zone contains particles with nearly the original size distribution. The third zone is composed of very small droplets. At about the time the leading edge of the flame reaches the third zone, the majority of the large droplets in zones one and two will begin to shatter into extremely fine droplets. Consequently, the combustion of these small droplets will cause an explosive heat release capable of supplying sufficient energy to enable the flame to overtake the shock front. The detonation wave is apparently sustained by the burning of these microdrops immediately behind the shock front. Therefore, Cramer concluded that some shattering mechanism is providing sufficient fuel vapor to sustain a detonation front.

From the previous discussion it is obvious that the drop size and the relative velocity between the droplet and the gaseous stream strongly influence the shattering phenomenon, and hence, the combustion mechanism. Burgoyne and Cohen<sup>17</sup> found that drops whose diameters were less than 10 microns behaved like a vapor, and drops with diameters greater than 40 microns burn individually.

Therefore, it is reasonable to expect the existence of two critical radii,  $r_{cr1}$  and  $r_{cr2}$ , such that the area for spray detonation is divided into three regions, see Table II.

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\*Also, the heterogeneous reaction assumption, non-volatile fuels, contributes to this conclusion.

\*\*The addition of the source term makes the equations difficult, if not impossible, to solve.

\*\*\*Cramer used two detonation tubes in his experiment, one transparent tube for taking pictures, and one steel tube for measuring  $p - t - x$  relations.

TABLE II

## CHARACTERISTIC REGIONS FOR SPRAY DETONATION

I	II	III
Drop diameters are small enough so that the classical gaseous detonation theory is applicable.	The droplets remain spherical and do not breakup. Therefore Williams' theory is valid, and detonation waves are sustained by both homogeneous and heterogeneous reactions.	Drop shattering is dominant. Williams' theory with the addition of a source term can be applied in principle. Detonation waves are stabilized by shattering phenomenon.

According to Burgoyne and Cohen,<sup>17</sup> it is reasonable to let  $r_{cr})_1 \approx 5\mu$ . The determination of  $r_{cr})_2$  is not so obvious because of its strong dependence on the relative velocity between the gas and droplets. A more detailed investigation of this parameter will be undertaken in our later studies of droplet breakup mechanisms.

The previous discussion implies that a droplet combustion driven shock front differs from gaseous detonation in at least four points; (a) the burning droplets add mass to the gaseous stream, (b) the droplet burning zone seems thicker, (c) the wave model is now a shock wave following by drop shattering and combustion, (d) a true detonation wave in sprays seems doubtful, however, flame propagation in this case is extremely fast and a violent reaction might be achieved.\*

Since the droplet shattering phenomenon plays an important role in detonations through sprays, it is felt that the mechanism of breakup, the breakup time, and the critical velocity and droplet size are fundamental parameters in the two-phase detonation process. It has been found that the disintegration phenomenon is due to the interaction between the droplet and the flow field behind the shock wave, and is not due to the interaction between the droplet and the shock front itself.<sup>14,18,19,20,21</sup>

The following types of breakup are known; (a) a vibrational phenomenon in which the oscillations of the droplet increase in amplitude until the droplet breaks into fragments, (b) shear type breakup wherein the relative velocity between the gas and the droplet causes a shear layer in the drop resulting in the formation of smaller droplets, (c) bag type breakup wherein the gaseous flow causes the drop to flatten and disintegrate. It appears that only the bag and shear type breakup will be of importance in the study of the propagation of de-

\*Personal correspondence with J. M. Pilcher, Battelle Memorial Institute, dated 26 July 1962.

tonation waves through sprays because the vibration type breakup is a relatively slow process. Schematics of these two important types of breakup are shown in Figs. 2a and 2b.

The studies concerning droplet shattering are continuing.

## B. STUDY OF HEAT TRANSFER IN THE ROTATING DETONATION ENGINE

As in conventional rocket motors, heat transfer to the walls of the rotating detonation engine is a key factor in determining the feasibility and design of various engine configurations. A preliminary study of the heat transfer problem has been made and is described below.

An analysis of the temperature distribution in the wall of the rotating wave engine breaks down into the problems of (1) determining the magnitude and time variation of the heat flux from the hot gases behind the detonation wave to the walls of the combustion chamber, and (2) computing the heat conduction in the chamber wall. A detailed discussion of these problems follows below.

### 1. Theoretical Model for Heat Flux

To determine the heat flux it is necessary to have some understanding of the processes which occur in the rotating wave engine. The rotating detonation is followed by a region of high pressure and high temperature gases where the heat flux will be a maximum. After the detonation passes, the combustion products expand through the annular nozzle of the wave engine and fresh fuel and oxidant enter the combustion chamber. During this process the temperature and pressure and consequently the heat flux decrease until at some distance behind the wave the heat flux becomes negligible. Since the flow described above is quite complex the simplified theoretical model described below has been adopted for the initial heat transfer calculations. In later more refined calculations this model will be improved.

It is assumed that the detonation wave is plane and moves past a flat plate and through a combustible mixture which is initially at rest as shown in Fig. 3. The velocity induced by the passage of the detonation results in the formation of a boundary layer. Pressure, temperature, and velocity behind the detonation are assumed constant and equal to the Chapman-Jouguet values for some distance  $x_p$  behind the wave. Beyond this point it is assumed that heat transfer will be negligible so that  $x_p$  represents a heat pulse width. Since the flow appears steady to an observer moving with the detonation wave a coordinate system which is fixed to the wave as shown in Fig. 4 has been adopted. In this coordinate system the wall moves with detonation velocity  $V_p$ .

## 2. Condition of Combustion Gases Outside the Boundary Layer and Behind the Detonation

To determine the velocity, pressure, and temperature of the fluid outside the boundary layer induced by the detonation, it is necessary to use the one dimensional conservation equations which are as follows:

$$\text{Mass:} \quad \rho_1 V_D = \rho_e V_e \quad (1)$$

$$\text{Momentum:} \quad p_1 + \rho_1 V_D^2 = p_e + \rho_e V_e^2 \quad (2)$$

$$\text{Energy:} \quad h_1 + \frac{V_D^2}{2} + Q = h_e + \frac{V_e^2}{2} \quad (3)$$

where  $h$  = enthalpy and  $Q$  is the heat released by the chemical reaction. In addition to the conservation equations it is assumed that the perfect gas approximation holds so that

$$p = \frac{\mathcal{R}}{\mathcal{M}} \rho T \quad (4)$$

where  $\mathcal{R}$  = universal gas constant  
 $\mathcal{M}$  = molecular weight.

Finally it is assumed that the wave in question is a Chapman-Jouguet detonation so that  $V_e$  is the local speed of sound, i.e.,

$$\frac{V_e^2 \mathcal{M}_e}{\gamma_e \mathcal{R} T_e} = M_e^2 = 1 \quad (5)$$

Equations (1) to (5) have been extensively discussed, e.g., see Morrison.<sup>23</sup> Unfortunately the heat released,  $Q$ , can only be computed by laborious iterative chemical equilibrium calculations. Fortunately both theoretical calculations and experimental measurements for various  $H_2$ - $O_2$  mixtures, which is the fuel oxidant combination used by the wave engine, have been made by Moyle<sup>24</sup> and so his results will be used to determine  $V_e$ ,  $T_e$ ,  $p_e$ , and  $\rho_e$ .

Moyle has calculated the ratio  $V_D/V_e$ , the detonation velocity  $V_D$ , and  $\mathcal{M}_e$ , the molecular weight of the equilibrium mixture behind the detonation, for various mixture ratios and initial conditions. Moyle's calculated values are in close agreement with experimental results. Assuming that  $\gamma_e$ , the ratio of specific heats behind the wave, has the value 1.22, which from Moyle's results ap-



pears to be valid for initial mixtures ranging from 78% H<sub>2</sub> by volume to 35% H<sub>2</sub> by volume, all quantities behind the detonation can be calculated.

For example for an initial H<sub>2</sub>-O<sub>2</sub> mixture of 60% H<sub>2</sub> by volume and an initial pressure and temperature of 1 atmosphere and 300°K it is found that

$$\begin{aligned} V_D &= 8620 \text{ ft/sec} & T_e &= 6322^\circ\text{R} \\ V_D/V_e &= 1.777 & \rho_e &= .0630 \frac{\text{lb mass}}{\text{ft}^3} \\ V_e &= 4851 \text{ ft/sec} \end{aligned}$$

### 3. Heat Flux Across the Boundary Layer

It has been assumed that the boundary layer formed by the detonation is turbulent throughout. The presence of a combustion zone within the detonation, which in itself tends to be turbulent,<sup>25</sup> provides a reasonable basis for this assumption.

To obtain an initial estimate of the heat flux Mirels'<sup>26</sup> analysis of the turbulent boundary layer behind a moving shock wave in air has been applied to the present problem. The free stream conditions in Mirels' analysis are constant as is the case in the model adopted for the present analysis. Using a Reynolds analogy Mirels developed the following relation between the heat flux  $q_w$  and shear  $\tau_w$  at the wall:

$$q_w = \frac{(h_r - h_w) \tau_w}{(V_D - V_e) \text{Pr}_m^{2/3}} \quad (6)$$

where  $h_w$  is the wall enthalpy, and  $\text{Pr}_m$  is the Prandtl number evaluated at some mean or reference condition within the boundary layer.  $h_r$  is the recovery enthalpy and is given by

$$h_r = h_e + \left( \frac{V_D}{V_e} - 1 \right)^2 \frac{V_e^2}{2} \text{Pr}_m^{1/3}. \quad (7)$$

For mean conditions in the boundary layer Mirels uses fluid properties based on Eckert's<sup>27</sup> mean enthalpy which is defined as

$$h_m = 0.5(h_w + h_e) + 0.22(h_r - h_e). \quad (8)$$

The shear stress at the wall,  $\tau_w$ , was determined by a solution of the momentum integral equation, which incorporates the moving wall boundary condition that differentiates the shock tube and conventional boundary layers. Mirels obtained the result

$$\frac{\tau_w}{\rho_e V_e^2} = .0460 \left( \frac{\theta}{\delta} \right) \left[ \phi \left( 1 - \frac{V_D}{V_e} \right) \frac{\delta}{\theta} \right]^{4/5} \left( \frac{V_D}{V_e} - 1 \right)^{3/5} \left( \frac{V_e}{V_e} \right)^{1/5} \left( \frac{1}{x} \right)^{1/5} \quad (9)$$

where

$$\phi = \left( \frac{\mu_m}{\mu_e} \right)^{1/4} \left( \frac{\rho_m}{\rho_e} \right)^{3/4},$$

$\nu_e$  is the kinematic viscosity in the free stream, and  $\delta$  and  $\theta$  are the boundary layer and momentum thicknesses respectively. The ratio  $\theta/\delta$  is given as a complicated function of  $h_r$ ,  $h_w$ ,  $h_e$  and  $V_D/V_e$  by Mirels; however, in the present case the formula

$$\frac{\theta}{\delta} = 0.317 \left[ 1 - \frac{V_D}{V_e} \right] \quad (10)$$

given by Hartunian,<sup>28</sup> et. al., for shock Mach numbers above 5.0 will be used. Mirels analytical results are in good agreement with the experimental results of Hartunian and this provides some assurance that the formulas above will yield reasonable results. The experimental results also fit the formula

$$St \sqrt[5]{Re} = 3.7 \times 10^{-2} \quad (11)$$

where

$$St = \text{Stanton No.} = \frac{q_w}{\rho_e (V_D - V_e) (h_r - h_w)}$$

$$Re = \text{Reynolds No.} = \frac{\rho_e (V_D - V_e) x}{\mu_e}$$

Throughout the work of Mirels and Hartunian equilibrium air properties are used. Since the boundary layer equations used by Mirels did not include a term for diffusive transport, it would appear that Mirels has assumed equilibrium flow with the Lewis number,  $Le = (CpD_p)/k$ , equal to unity, although this fact is never explicitly stated. In the preliminary calculations described below the effects

of dissociation have been ignored. Computations based on equilibrium properties of the combustion gases will be made in the immediate future.

For the 60%  $H_2$  mixture considered above, Eqs. (6) through (10) yielded the following result for the heat flux:

$$q_w = 1986 \left( \frac{1}{x} \right)^{1/5} \frac{\text{Btu}}{\text{ft}^2 \text{sec}} \quad (12)$$

where  $x$  is distance behind the detonation in feet. Moyle's<sup>24</sup> results were used for the composition behind the detonation wave and in the present calculation it was assumed that this composition remains fixed throughout. Enthalpies were obtained from NBS tables,<sup>29</sup> and transport properties of the gas mixture behind the detonation were calculated using the charts and formulae given in Barrere.<sup>30</sup>  $q_w$  is infinite at the foot of the detonation where  $x = 0$  because of the leading edge singularity of the boundary layer; however, the total or integrated heat flux remains finite.

In general

$$q_w = Kx^{-1/5}$$

where  $K$  depends on the properties of the detonation. The total heat flux,  $Q_w$ , over a pulse of length  $x_p$  is

$$Q_w = K \int_0^{x_p} x^{-1/5} dx = \frac{5}{4} K x_p^{4/5} \text{Btu} \quad (13)$$

for the unit width of the flat plate.  $\bar{q}_w$ , the average heat flux, is given by

$$\bar{q}_w = \frac{Q_w}{x_p} = \frac{5}{4} K x_p^{-1/5} \frac{\text{Btu}}{\text{ft}^2 \text{sec}} . \quad (14)$$

A stationary observer sees a periodic free stream variation of heat flux of period  $\tau$ . If there are  $n$  equally spaced waves rotating in an annulus of diameter  $D$  then

$$\tau = \frac{\pi D}{n V_D} . \quad (15)$$

The ratio  $c$  of the pulse width to the wavelength will be given by

$$c = \frac{nx_p}{\pi D} = \frac{t_p}{\tau} \quad (16)$$

where  $t_p$  is the pulse duration, and is related to  $x_p$  by

$$x_p = V_D t_p = c \tau V_D. \quad (17)$$

During the passage of each detonation a stationary observer will see a time varying heat flux  $q_w(t)$  given by

$$q_w(t) = K(V_D t)^{-1/5}. \quad (18)$$

To simplify the conduction problem discussed below the actual variation (18) has been replaced by a series of square pulses of width  $c\tau$  and amplitude  $\bar{q}_w$  as shown in Fig. 5.

The overall combustion chamber cooling problem depends on the average heat transfer,  $\bar{q}_w$ , to the walls of the chamber per unit time per unit area.  $\bar{q}_w$  is given by

$$\bar{q}_w = c \bar{q}_w = \frac{5}{4} K c^{4/5} \left( \frac{n}{\pi D} \right)^{1/5}. \quad (19)$$

Equation (19) shows that the dimensionless heat pulse width,  $c$ , is of crucial importance in determining the overall chamber heat transfer. If Eq. (16) is combined with (19) the following expression for  $\bar{q}_w$  is obtained:

$$\bar{q}_w = \frac{5}{4} K \frac{nx_p^{4/5}}{\pi D}. \quad (20)$$

Equation (20) shows that for a given pulse width  $x_p$ ,  $\bar{q}_w$  varies directly with the number of detonations rotating about the chamber and inversely with the diameter of the annular combustion chamber. Either  $c$  or  $x_p$  depends on the complex flow behind the detonation, which is presently under investigation.

As a specific example, the case of the 7 in. ID, 8 in. OD, 100 lb thrust experimental engine operating with a 60%  $\phi_2$  mixture has been considered. It has

been assumed that  $c = 1/3$ . Pressure traces obtained during the original tests of this engine indicate that this assumption for  $c$  is quite reasonable for the high pressures behind the detonation seem to persist for only about  $1/3$  of each cycle. From Eqs. (12), (14), (15) and (17) it follows that

$$\tau = 2.28 \times 10^{-4} \text{ sec} = 228 \text{ micro sec}$$

$$\bar{q}_w = 2700 \text{ Btu/ft}^2\text{sec} = 18.8 \text{ Btu/in.}^2 \text{ sec}$$

and from Eq. (19) it follows that with  $c = 1/3$

$$\bar{q}_w = 900 \text{ Btu/ft}^2\text{sec} = 6.25 \text{ Btu/in.}^2\text{sec} .$$

$\bar{q}_w$  is of the same order of magnitude as the heat flux near the throat of a conventional rocket motor.

The calculations of  $\bar{q}_w$  involves numerous approximations. First it should be mentioned that heat flux due to radiation has been neglected. Experience with conventional rocket motors indicates that radiation may increase the heat flux 10-20%. Conditions behind the detonation wave are not constant, but rather because of expansion through the nozzle and admission of fresh unburnt fuel and oxidant there will be a rapid drop in temperature and pressure. In ignoring this factor the calculations above are quite conservative. Consideration of the fact that the dissociated combustion products may recombine near the cool wall of the combustion chamber may cause some increase in  $\bar{q}_w$ .

The combustion chamber surface temperature depends upon the conduction within the chamber walls and this problem is discussed below.

#### 4. Heat Conduction in the Solid Wall

To solve the combustion chamber heat transfer problem it is necessary to consider the conduction through the cylindrical inner and outer walls of the chamber as shown in Fig. 6. On one surface of each cylinder the boundary condition consists of a series of equally spaced heat pulses moving past the surface with velocity  $V_p$ . The other surface is in contact with coolant. An exact solution of this two dimensional unsteady conduction problem is very difficult. It has been found possible to make two simplifications which greatly simplify the analysis.

If  $d_i/R_c \ll 1$  and  $d_e/R_c \ll 1$ , where  $R_c$  is average combustion chamber radius and  $d_i$  and  $d_e$  are inner and outer wall thickness, then the cylindrical walls can be replaced by an infinite flat plate with heat pulses moving past one side and coolant on the other side. If the width  $x_p$  of the heat pulse is sufficiently

great and the period  $\tau$  of the heat pulse is sufficiently small then the effects of the periodicity of the heat pulse will be confined to a thin region near the surface of the flat plate. In such case the work of Jaeger<sup>31</sup> indicates that for purposes of determining the maximum surface temperature two dimensional effects can be neglected. Thus one can replace the combustion chamber conduction problem by the conduction through a one dimensional flat plate with periodic surface conditions.

The above approximation has been applied by Phillips<sup>32</sup> to the problem of heat transfer from a moving arc to an electrode, and his results are directly applicable to the present problem. Phillips determined the temperature within an infinite flat plate of thickness  $\ell$  with a periodic heat flux on one surface and heat transfer across a coolant film on the other surface, as shown in Fig. 7. Phillips obtained the solution of the heat equation

$$\frac{\partial^2 T}{\partial y^2} - \frac{1}{\alpha} \frac{\partial T}{\partial t} = 0 \quad (21)$$

subject to the boundary conditions

$$\begin{aligned} -k \frac{\partial T}{\partial y} &= \bar{q}_w; \quad m\tau \leq t \leq (m+c)\tau \\ -k \frac{\partial T}{\partial y} &= 0; \quad (m+c)\tau \leq t \leq (m+1)\tau \end{aligned} \quad (22)$$

$$m = 0, 1, 2, 3$$

at  $y = 0$

and

$$-k \frac{\partial T}{\partial y} = h(T - T_0); \quad y = \ell \quad (23)$$

where  $\alpha$  is the thermal diffusivity,  $k/\rho c_s$ ,  $h$  is the coolant film coefficient,  $k$  the conductivity, and  $c_s$  the specific heat. Equation (22) is an analytical representation of square pulse heat flux shown in Fig. 5. Initial conditions are

$$T(y, 0) = T_0 \quad (24)$$

The solution of the problem above consists of a steady state part and a transient part which dies out as  $t \rightarrow \infty$ . In the present case only the steady

state value of the surface temperature is of interest, and is given by

$$(T_s - T_0) = \frac{2\bar{q}_w l}{k} \sum_{n=1}^{\infty} p_n \left[ \left\{ 1 - \exp\left(-\frac{\beta_n^2 \alpha}{l^2} (t - m\tau)\right) \right\} + \exp\left(-\frac{\beta_n^2 \alpha}{l^2} (t - m\tau)\right) \left\{ \frac{1 - \exp\left[\frac{c\tau\beta_n^2}{l^2}\right]}{1 - \exp\left[\frac{\tau\alpha\beta_n^2}{l^2}\right]} \right\} \right] - \frac{\bar{q}_w c}{h} (1 + h'l), \quad (25)$$

where  $\beta_n$  are eigenvalues determined from the equation

$$\beta_n \tan \beta_n = h'l \quad (26)$$

and

$$p_n = \frac{[\beta_n^2 + (h'l)^2]}{\beta_n^2 [\beta_n^2 + h'l(1 + h'l)]} \quad (27)$$

where  $h' = h/k$ . Equation (25) is valid only during intervals

$$m\tau \leq t \leq (m + c)\tau$$

$$m = 0, 1, 2, 3, \dots$$

when  $q = \bar{q}_w$ . Only this part of the solution is of interest for it is during this interval that the surface temperature reaches its maximum value. The maximum surface temperature, which occurs at the end of the square heat pulse is according to (25) given by

$$(T_{s_{\max}} - T_0) = \frac{2\bar{q}_w l}{k} \sum_{n=1}^{\infty} p_n \left\{ \frac{1 - \exp\left[-\frac{c\alpha\tau\beta_n^2}{l^2}\right]}{1 - \exp\left[-\frac{\alpha\tau\beta_n^2}{l^2}\right]} \right\} - \frac{\bar{q}_w c}{h} (1 + h'l), \quad (28)$$

while the minimum surface temperature, which occurs at the start of the heat pulse is

$$(T_{s_{\min}} - T_0) = \frac{2\bar{q}_w l}{k} \sum_{n=1}^{\infty} p_n \left[ \frac{1 - \exp\left[\frac{\alpha\tau\beta_n^2}{l^2}\right]}{1 - \exp\left[\frac{\tau\alpha\beta_n^2}{l^2}\right]} \right] - \frac{\bar{q}_w c}{h} (1 + h'l) \quad (29)$$

The amplitude of the surface temperature fluctuation, i.e.,  $T_{s\max} - T_{s\min}$  depends upon the dimensionless pulse width  $c$  and the factor  $\alpha\tau/\ell^2$ . For  $c = 1$ , i.e., continuous heat flux,  $T_{s\max} = T_{s\min}$  as is to be expected. From (26) it is readily shown that

$$\beta_n < (2n - 1) \frac{\pi}{2}$$

so that

$$\frac{\alpha\tau}{\ell^2} \beta_n^2 < (2n - 1)^2 \frac{\pi^2}{4} \frac{\alpha\tau}{\ell^2} \quad (30)$$

From Eqs. (28) and (29) it thus follows that if  $(\alpha\tau/\ell^2) \ll 1$  then surface temperature fluctuations also will be quite small. This is evident if the exponentials in (28) and (29) are expanded so that the following equation is obtained:

$$\frac{T_{s\max} - T_{s\min}}{\frac{(c\bar{q}_w\ell)}{k}} = \frac{2}{c} \sum_{n=1}^n p_n \left[ \frac{\alpha\tau}{\ell^2} \beta_n^2 c(1 - c) + O\left(\frac{\alpha\tau}{\ell^2} \beta_n^2\right) \right] \quad (31)$$

where  $c\bar{q}_w\ell/k$  is the temperature drop through the plate if the heat pulses are replaced by a steady heat flux  $c\bar{q}_w$ . The series (31) is only carried to  $n_\ell$  such that for  $n \leq n_\ell$ ,  $\alpha\tau\beta_n^2/\ell^2 \ll 1$ . For  $n > n_\ell$  the series expansion of the exponentials in (28) and (29) no longer will be valid. If  $n_\ell$  is sufficiently large the portion of the series (28) and (29) with  $n > n_\ell$  will be negligible since  $p_n \sim 1/\beta_n^2$ . In that case (31) provides a good approximation of the amplitude of the temperature fluctuation.

A specific example will be considered. Assuming, somewhat arbitrarily, that it is necessary that

$$\frac{\alpha\tau\beta_n^2}{\ell^2} < 0.01$$

for (31) to be valid, and that  $n_\ell$  is such that  $\beta_n^2 \approx 10$  it follows that the condition

$$\frac{\alpha\tau}{\ell^2} < 0.01$$

must be satisfied. In such case the dimensionless temperature fluctuation given by (31) will be approximately .025 since



$$P_n \sim \frac{1}{\beta_n^2} ; n_l \sim 2 .$$

Thus the surface temperature fluctuation is essentially negligible. Now assuming a brass plate with  $\alpha = 3.6 \times 10^{-4} \text{ ft}^2/\text{sec}^2$  and using the value of  $\tau$  found in Section 3,

$$\alpha\tau = 8.2 \times 10^{-8} \text{ ft}^2$$

so that for

$$l \geq 2.87 \times 10^{-3} \text{ ft} = .034 \text{ in.}$$

surface temperature fluctuations will be negligible.

Phillips analysis leads to the conclusion that in most cases the time varying heat flux may be replaced by an average steady value. The only exception to this result will arise if cooling system designs with very thin chamber walls are considered. If temperature fluctuations are neglected the surface temperature is given by

$$T_s - T_o = \bar{q}_w \left( \frac{l}{k} + \frac{1}{h} \right) \quad (32)$$

where the film coefficient  $h$  reflects the cooling system design.

Assuming a steady heat input, the time required for the combustion chamber surface of an uncooled engine to reach the melting temperature was computed. For this purpose the solution for the semi-infinite solid with a constant surface heat flux, which is given by Carslaw and Jaeger,<sup>33</sup> was used. Using the average heat flux

$$c\bar{q}_w = \frac{1}{3} (2700) = 900 \text{ Btu/ft}^2\text{sec}$$

calculated in Section II-3 it was found that the surface temperature rises from  $540^\circ\text{R}$  to  $2160^\circ\text{R}$ , the melting temperature of brass, in 2 seconds.

## 5. Conclusions and Problems to be Studied in Future Work

An initial estimate of heat transfer in the rotating detonation engine has been made. Two key conclusions are (1) Heat flux is of the same order of magnitude as in conventional rocket motors. (2) The amplitude of combustion chamber

surface temperature fluctuations are in most cases sufficiently small that the time varying heat flux can be approximated by an average steady value. The only exception to this result arises if combustion chamber walls are thin.

From the work completed to date it would appear that the following problems should be investigated:

- a. Effect of dissociation and recombination upon the heat flux behind the detonation wave.
- b. Detailed study of expansion behind the detonation in order to determine the dimensionless pulse width  $c$ .
- c. Analysis of turbulent boundary layer behind the detonation with variable free stream properties.
- d. The effect of mixture ratio, and fuel and oxidant pressure and temperature upon heat flux should be established.
- e. Preliminary design and evaluation of chamber cooling systems for steady operation should be made.
- f. An estimate of radiative heat transfer should be made.

Finally it should be remarked that experimental heat flux measurements are required to determine the validity of the approximate heat transfer analysis.

### III. EXPERIMENTAL STUDIES

#### A. THE GASEOUS 100 LB THRUST MOTOR

Although the proposal and contract did not specify the testing of the existing 100 lb thrust rocket motor, it was deemed very advantageous and important to do so. The reasons for the decision are: (a) experimental tests using the 100 lb thrust motor will be quite useful in evaluating some of the instrumentation desired for the 1000 lb thrust motor tests, (b) auxillary performance data can be obtained from these tests; performance data for the 100 and 1000 lb thrust rocket chamber being the end result of the test program, (c) through the use of the present 100 lb thrust chamber, a better understanding of the connection between the characteristic length and the transient chamber pressure can probably be obtained, and (d) it is possible that some cryogenic experience could also be obtained during the tests on this 100 lb thrust unit. This part of the program is essential because the knowledge gained from any of the areas mentioned would greatly facilitate the design and testing of the larger thrust chamber. In addition, there is the advantage of having the performance data for both the 100 lb and 1000 lb thrust units.

It is expected that this particular phase of the experiment will be continued until testing of the 1000 lb thrust unit begins.

The 100 lb thrust chamber has been modified so that a variation in the characteristic length, can be easily obtained, either by lengthening the thrust chamber or by changing the dimension of the throat. The variation of this parameter,  $L^*$ , should show the connection between it, the chamber pressure characteristics, and the heat transfer to the wall. Also, the modification of the 100 lb thrust motor now makes it possible to replace an injector face quite easily, Fig. 8.

The instrumentation for measuring the wall temperature and chamber pressure is complete and consists of a high frequency response, 1 microsecond rise time, quartz crystal pressure transducer and a high frequency response, 10 microsecond rise time, film thermocouple. The recording system compatible with this instrumentation is presently under consideration and consists of a multi-channel oscilloscope used in conjunction with a high speed streak camera. This system, similar to the General Dynamics Electronic TARE system,\* can be used either as a high frequency response oscillograph or the motion of the film can be rotated 90° for the analysis of one particular physical phenomenon. After a preliminary investigation, it is felt that this system offers more advantages per dollar than any other system previously considered.

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\*The General Dynamics Electronics TARE system was developed under an Air Force contract.

## B. TEMPERATURE AND PRESSURE EFFECTS ON HYDROGEN-OXYGEN DETONATION VELOCITIES

Since the propellants could be injected into the combustion chamber in a supercritical state, it was considered necessary to investigate the effect of temperature and mixture, e.g., Mach number of propagation and the pressure and temperature ratios across the detonation wave, near the oxygen vapor dome. The design of an experiment to do this was greatly facilitated by Moyle,<sup>24</sup> who showed that the detonation velocity for a mixture in a coiled detonation tube is the same as that for a straight tube if the experiment is carried out at room temperature, Fig. 9.

Moyle obtained the detonation velocities for hydrogen-oxygen mixtures at standard atmospheric pressure and a temperature range of approximately 160°K to 500°K. It is the intent of these experiments to obtain data for pressures up to 51 atmospheres and temperatures in the range of 78°K-300°K. The experimental technique to be used is similar to that employed by Moyle although it will be more versatile. Moyle has suggested\* that some device be included in the experiments for removing the combustion products before the water vapor has time to freeze and foul the ionization probes which are employed to determine velocity. Therefore, a purge system using pure gaseous nitrogen was installed, and will be used immediately following each detonation. This nitrogen purge system, used in conjunction with a vacuum pump, to sublime small quantities of ice, should be able to maintain unfouled ionization probes without removing the coiled detonation tube from the liquid nitrogen bath. If these measures for avoiding ice in the detonation tube are not successful, it will make the operation of the detonation tube much more tedious. On the other hand, if the nitrogen purge produces the desired results, the two premixed reservoirs and the precooling coil will enable the experiments to be conducted quite efficiently. The detection of shorted ionization probes fouled by water droplets or ice crystals will be accomplished by a thyatron trigger circuit that will restore immediately after the passage of the detonation wave, Fig. 10, if the probes are not fouled. A drawing of the ionization probes to be used is shown in Fig. 11.

A schematic of the experimental setup to be used is shown in Fig. 12 while a photograph of part of the actual setup is shown in Fig. 13. The particular mixture to be tested for detonation velocity is stored in one of two reservoirs. The mixture ratio is determined by introducing the hydrogen and oxygen to the reservoir to a predetermined partial pressure. For each experiment some of this mixture is introduced into the coiled detonation tube which is immersed in a liquid nitrogen bath. Adjustable pressurization of the nitrogen bath yields the variable temperatures desired for the hydrogen-oxygen mixture. When the temperature of the mixture has stabilized to the desired value, the mixture is spark ignited and a detonation wave propagates through the coiled tube. Ionization probes in the wall detect the wave and activate a time interval counter so that the time to traverse a known distance (and hence the detonation velocity) is readily determined.

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\*Personal communication.

A straight detonation tube has been included in the experimental setup as a double check on the actual mixture ratio in the reservoir. Periodically a sample will be drawn from the reservoir and admitted to the straight detonation tube. This mixture will be detonated at standard pressure and temperature and the velocity measured. The detonation velocity of hydrogen-oxygen at standard temperature and pressure is sensitive to mixture composition and its' variation is well known. Hence, the velocity measurement affords a good check on the actual mixture ratio.

Except for the liquid nitrogen bath, another time interval counter, and a combustion gas exhaust system, the experimental setup is virtually completed. Preliminary experiments have already been run on the straight detonation tube and satisfactory results obtained.

### C. DETONATION THROUGH HETEROGENEOUS, LIQUID-GAS MEDIA

Since the propellants can be injected into the rocket combustion chamber in the liquid state, it appears that a two-phase (liquid-gas) detonation will occur in this situation rather than a gaseous detonation process. Therefore, problems concerning the existence, structure, velocity, and stability of detonation waves in sprays are important factors to be considered in determining the feasibility of the rotating detonation wave rocket motor.

A detonation tube design represented by Fig. 14, has been completed and has been let out for bids. The design seems versatile enough to carry out an extensive study of the characteristics of detonation waves in sprays. The detonation tube is designed so that it can either be charged with a high pressure saturated mixture of gas and then be rapidly cooled by an expansion process so that droplets form and grow or the droplets can be sprayed into the detonation tube much like Cramer<sup>12</sup> did in some of his work. The exact method of attack has not yet been finalized and it appears that possibly both methods will be used, since each possess unique advantages. The test section is provided with windows and ports for instrumentation and injectors throughout its length. In the beginning a detonation driver will suffice to transmit a shock into the test section which may or may not develop into a detonation. The design of this driver section will be sufficiently flexible to allow for the insertion of many spark plugs so that combustible gases can be burned at constant volume rather than by detonation if so desired. Progress and experience will dictate which is the more advantageous for the problem at hand.

A small conventional schlieren system will be used to study the structure of the detonation wave. The velocity, and hence stability, will be determined either by photographic means or by pressure probes in the wall of the detonation tube. The latter method will probably be used in the beginning followed by the photographic method using the Beckman and Whitley high speed framing camera, provided the detonation velocity is of the proper order of magnitude.

Experiments will be conducted over a range of Webber numbers and Reynolds numbers and attempts made to photograph the drops. Information obtained from this phase along with considerations of the theory as discussed earlier should be of great value in predicting the stability of detonation in heterogeneous media. This is certainly an important consideration if storable propellants were to be used in a rotating detonation wave rocket motor.

#### D. GEOMETRICAL TESTS

The effect of various degrees of one dimensional relief and the effect of curvature (pressure gradient effects) on the propagation characteristics of a detonation wave must be known before an intelligent design of an experimental rotating detonation wave rocket motor can be made.

A sketch of the experimental apparatus necessary to accomplish this study is shown in Fig. 15. The curved section of this system is designed so that the shape and velocity of the detonation wave can be determined optically. The experimental system now being constructed will only be capable of checking the effect of one radius of curvature, namely a radius of curvature equal to that of the present 100 lb thrust chamber. However, sufficient versatility shall be designed into this experiment, at some latter date, to enable a parametric study to be conducted. This experimental apparatus, and other possible ones to follow, have the facility for ascertaining the effect of various degrees of one dimensional relief on the detonation characteristics of a hydrogen-oxygen mixture. That is, a means of venting the high pressure gases behind the detonation can easily be incorporated.

A shadowgraph system, as indicated in Fig. 16, is expected to have sufficient sensitivity to observe the macroscopic properties of the detonation waves' interaction with the curved wall.

In addition to the detonation wave shape and velocity in the curved section, the detonative wave velocity in the straight detonation driver section shall also be recorded. This procedure will furnish data for double checking the mixture ratio obtained by partial pressure mixing.

All phases of this experiment are active at the present time and completion of the setup is expected sometime in the month of September.

#### IV. STUDY PLANS FOR THE NEXT QUARTER

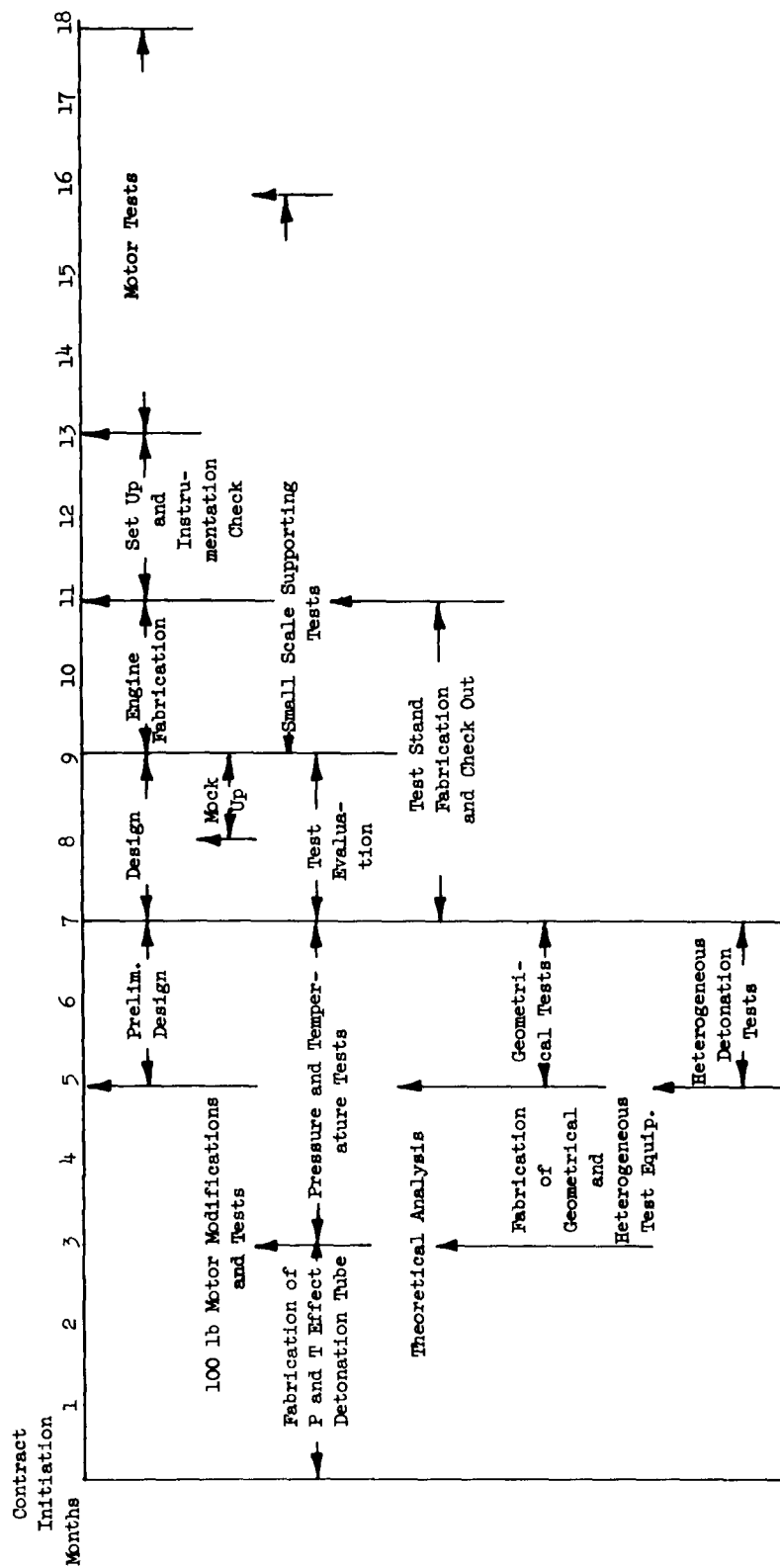
In addition to the continuation of the two theoretical studies described in this report, a study shall be made to obtain a simplified model for predicting the instantaneous chamber pressure in the rotating detonation wave combustion chamber. This will allow prediction of thrust characteristics and how this is influenced by the presence of tangential gas velocities.

The instrumentation on the experiments described in this report will be completed during the next quarter.

Preliminary tests on the 100 lb thrust rotating detonation wave engine will begin utilizing new temperature and pressure sensing devices and new recording methods. In addition to this, data concerning the effects of pressure, temperature, pressure gradient, relief, and droplets on the detonation characteristics will be obtained.

A representation of the proposed work schedule on this program is included.

PROPOSED WORK SCHEDULE  
(Revised 8 August 1962)





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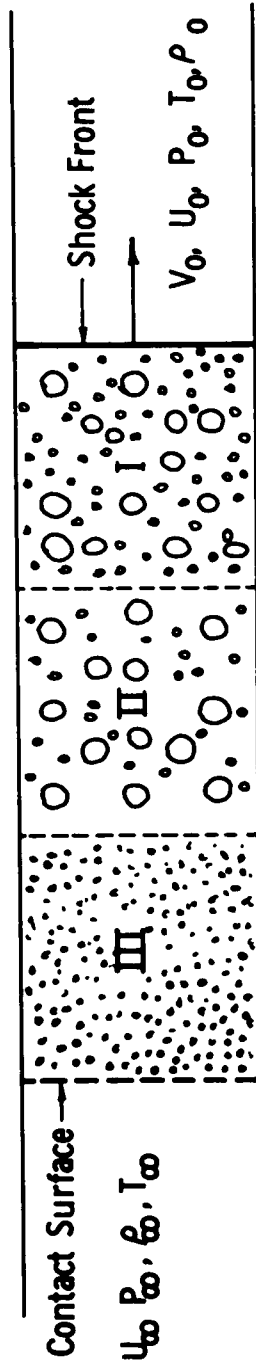
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At Time  $t$

- Zone I has a relatively high number density and the fine particles are burning.
- Zone II has approximately the original particle distribution and the fine particles are burning.
- Zone III is filled with micron-droplets produced by shattering.

<p>At time <math>t + \Delta t</math></p> <p>Contact Surface <math>\rightarrow</math></p> <p><math>U_\infty, \rho_\infty, T_\infty</math></p>	<p>The shattering produced micron-droplet mist begins to burn.</p>	<p>The large droplets in these regions are shattered into micron-droplets in the burning atmosphere. This results in a momentary explosive heat release capable of sustaining the detonation front.</p>	<p>Shock Front</p> <p><math>V_0, U_0, P_0, T_0, \rho_0</math></p>
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Fig. 1. Schematic burning process in spray detonation.

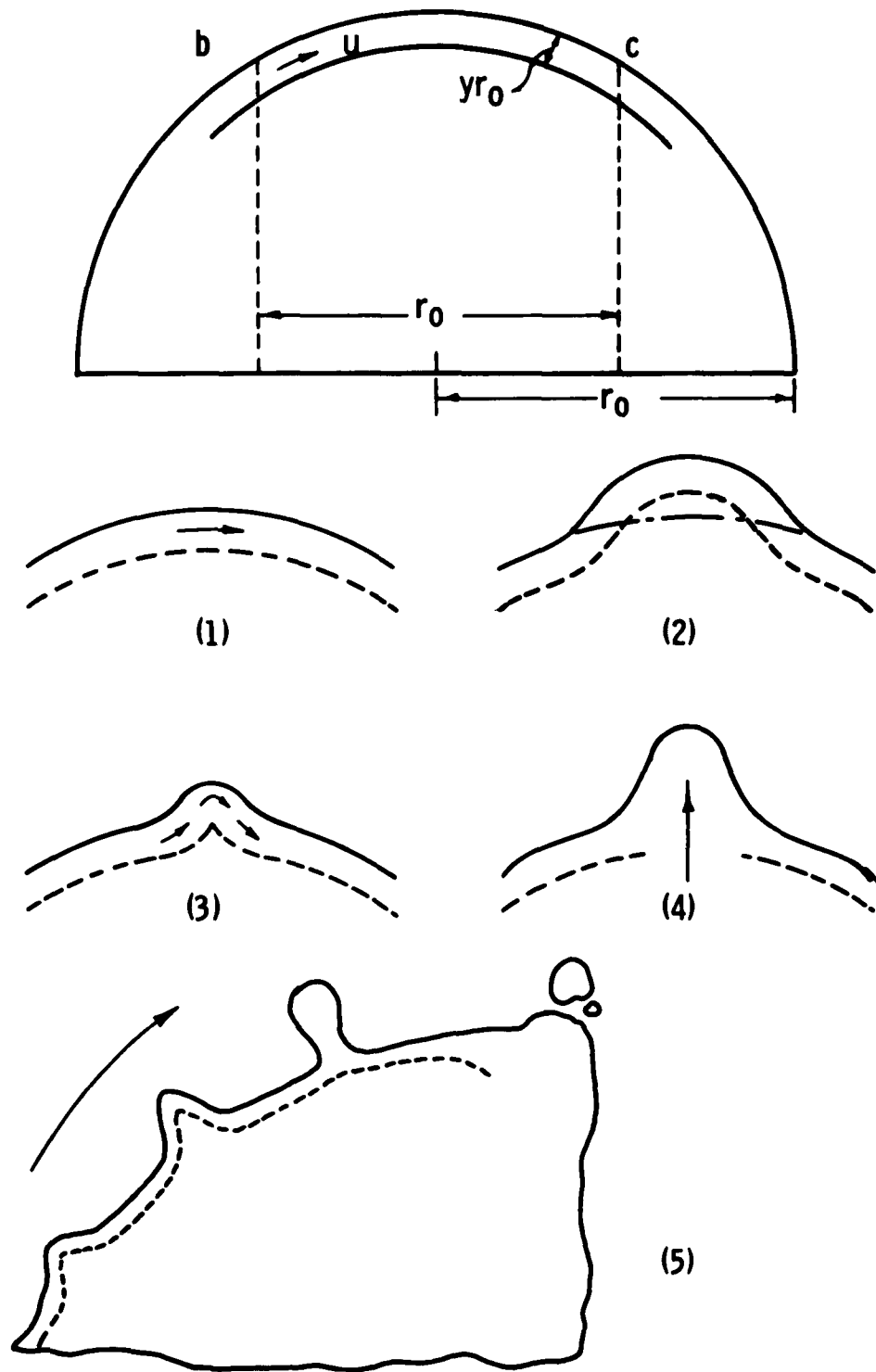


Fig. 2a. Schematic process for shear type breakup.

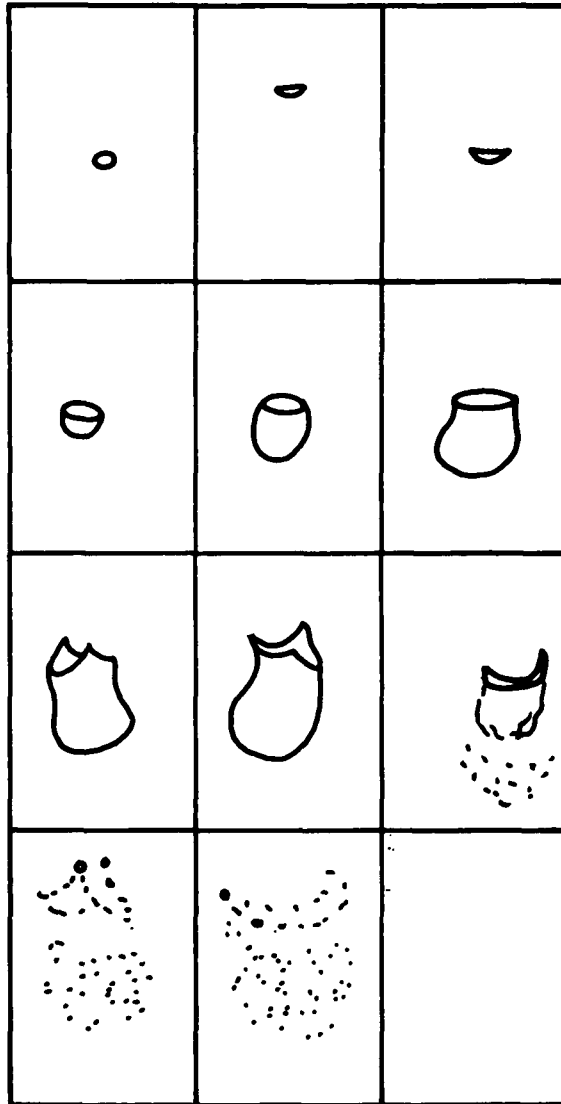


Fig. 2b. Schematic of bag type breakup.

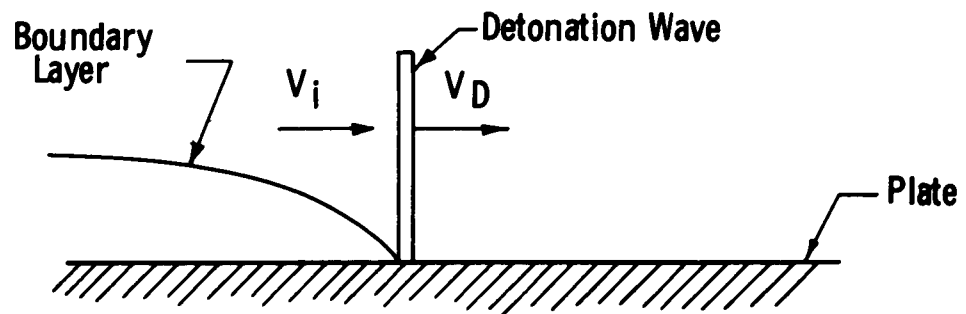


Fig. 3. Detonation moving over a flat plate.

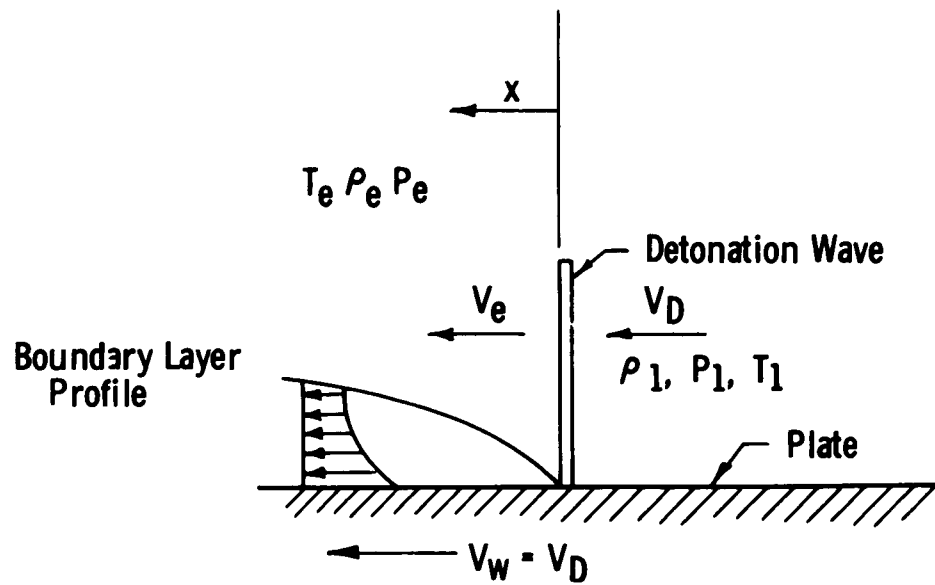


Fig. 4. Coordinate system for heat transfer analysis.



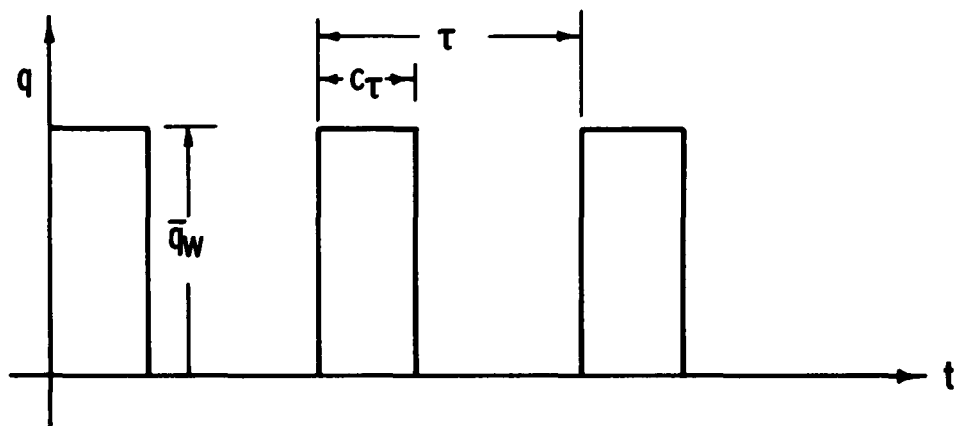


Fig. 5. Square pulse approximation of surface heat flux.

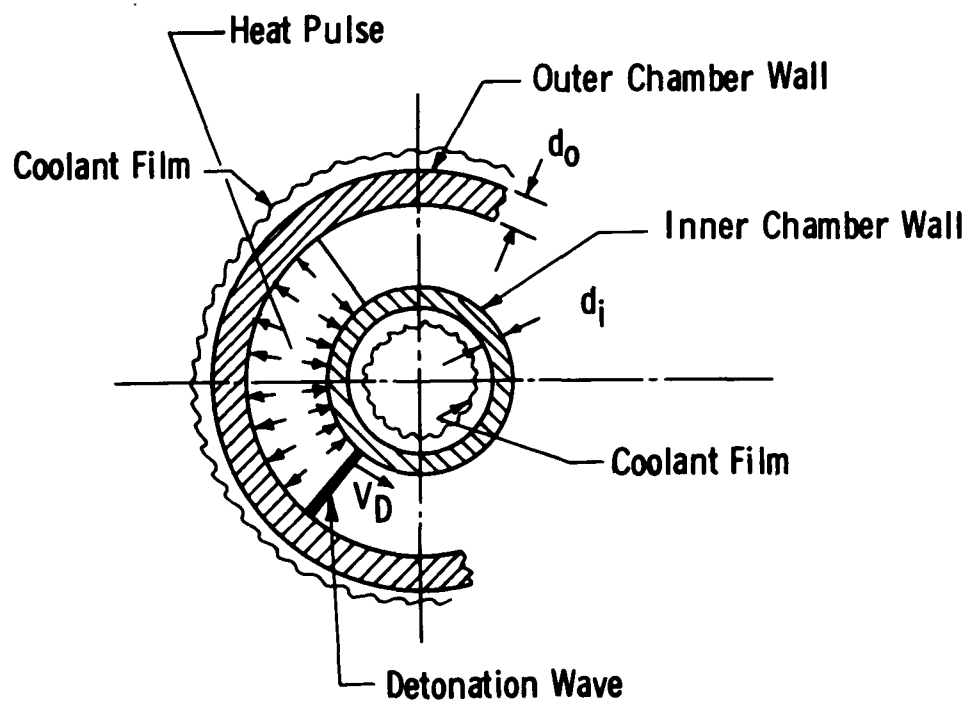


Fig. 6. Theoretical heat transfer model—rotating detonation wave engine.

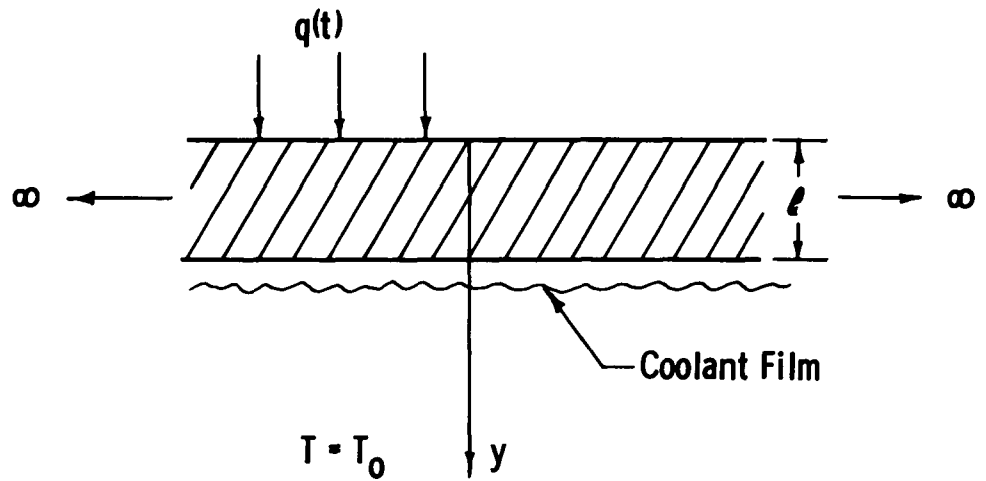


Fig. 7. One dimensional conduction model.

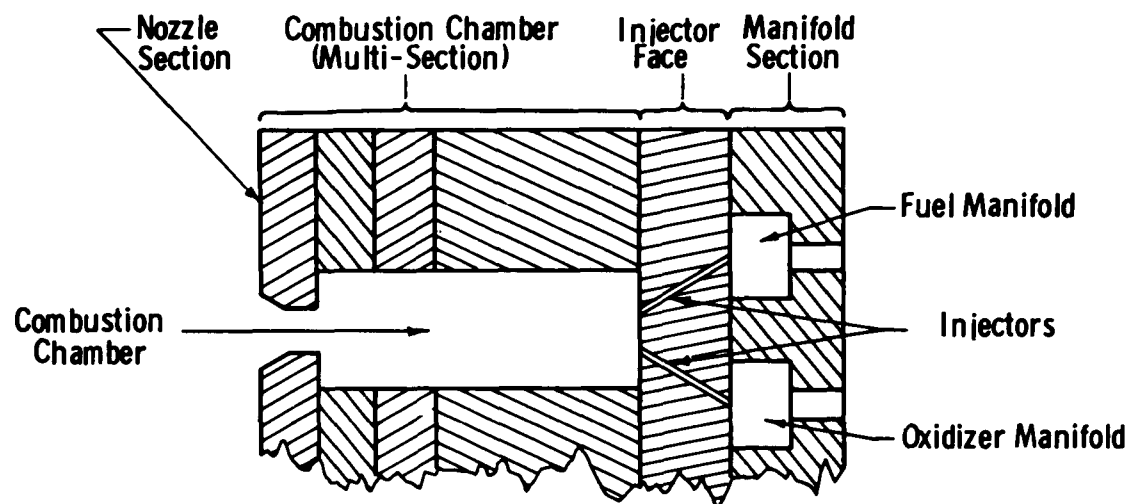


Fig. 8. Modified 100 lb thrust rotating detonation wave motor.

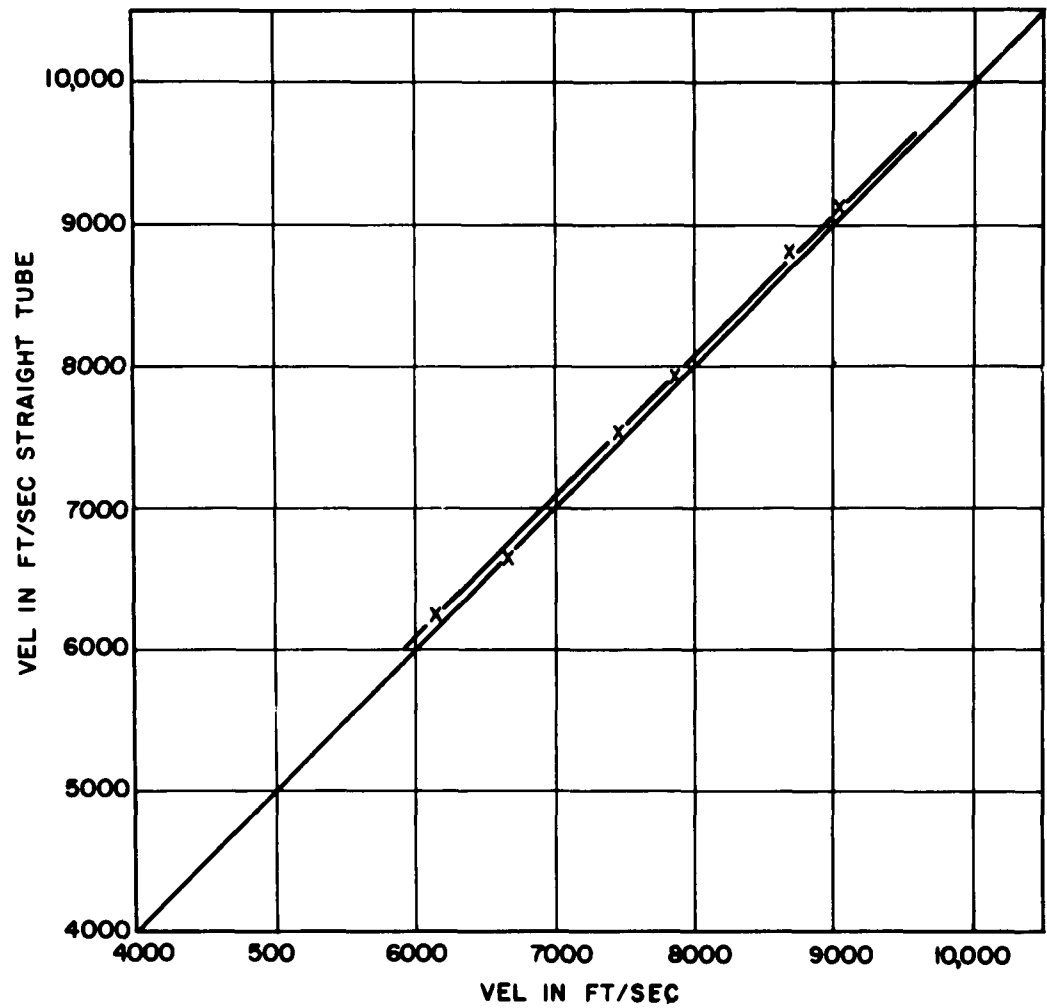


Fig. 9. Comparison of velocity of detonation in the coil with velocity in straight tube.

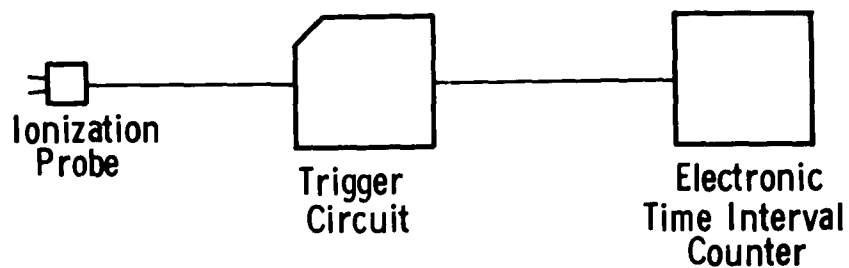


Fig. 10. Velocity measuring instrumentation of the pressure and temperature effects experiment.

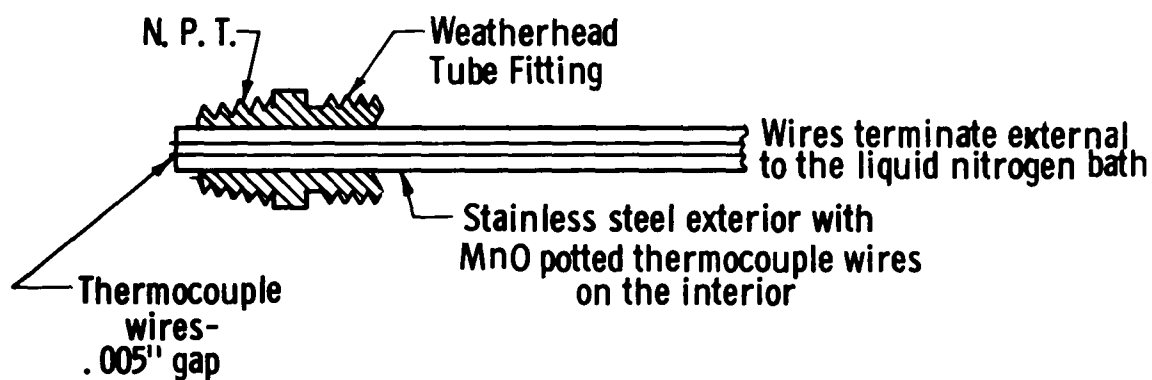


Fig. 11. Details of the ionization probe.

R - Regulator

⊗ - Shut-Off Valves

N - Metering Valves

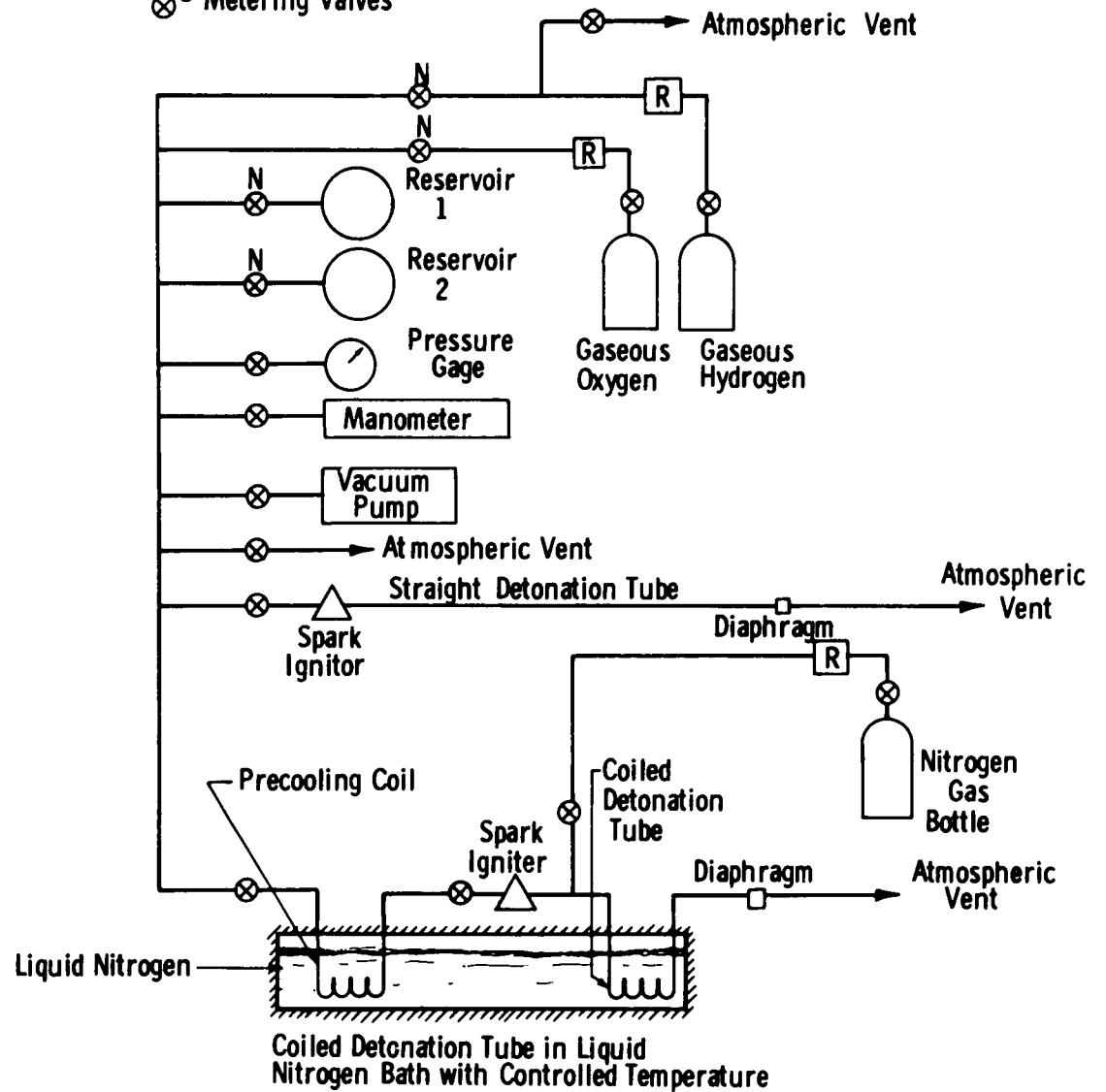


Fig. 12. Detonation wave propagation velocity testing system.



Fig. 13. Photograph of experimental setup for detonation velocity measurements.

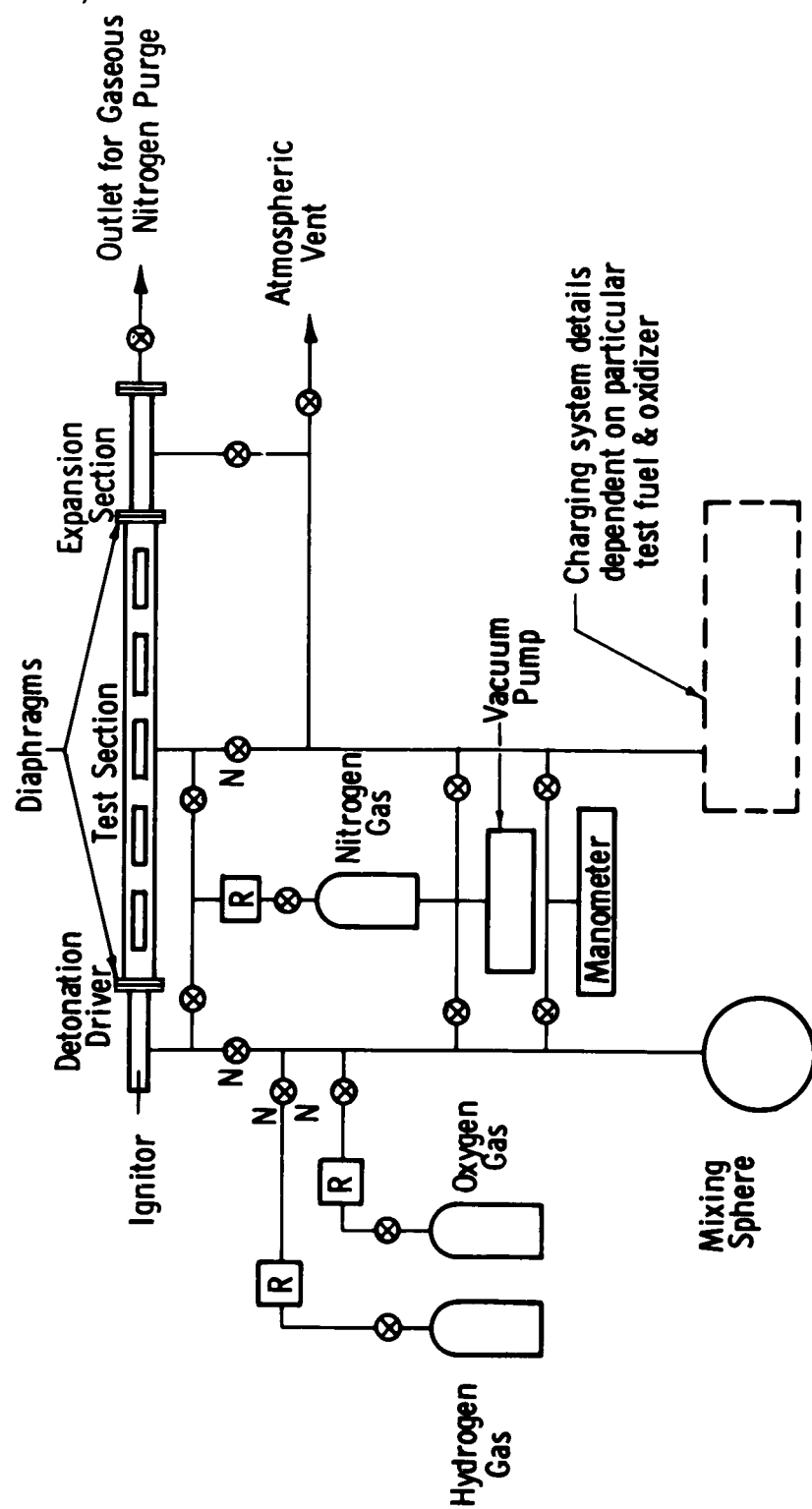


Fig. 14. Experimental setup for detonation in heterogeneous media.

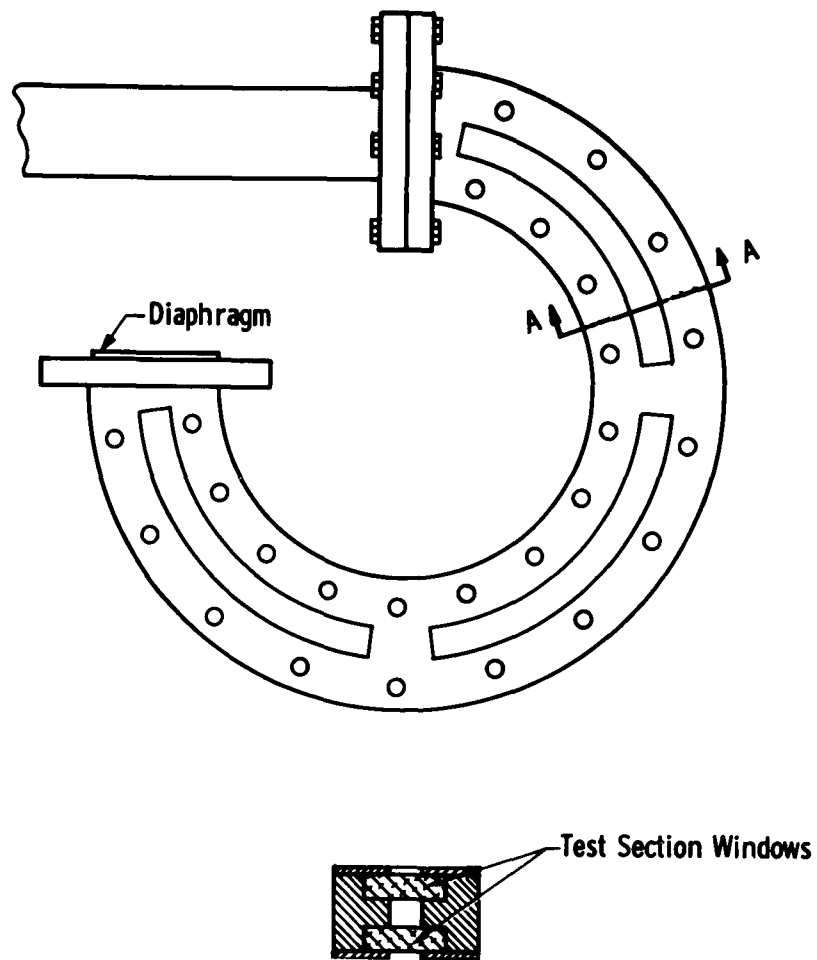


Fig. 15a. Test section for detonation in a curved channel.

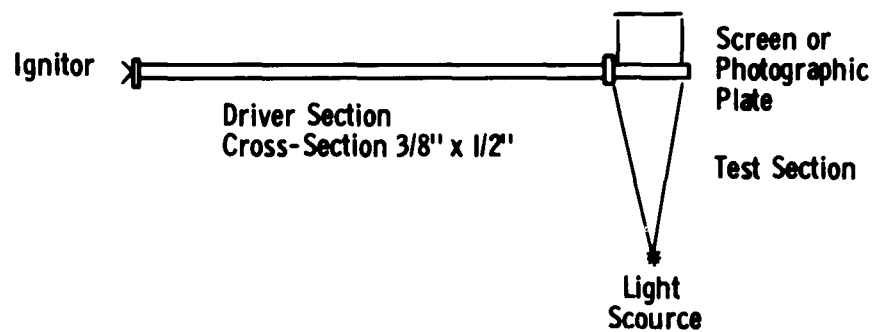
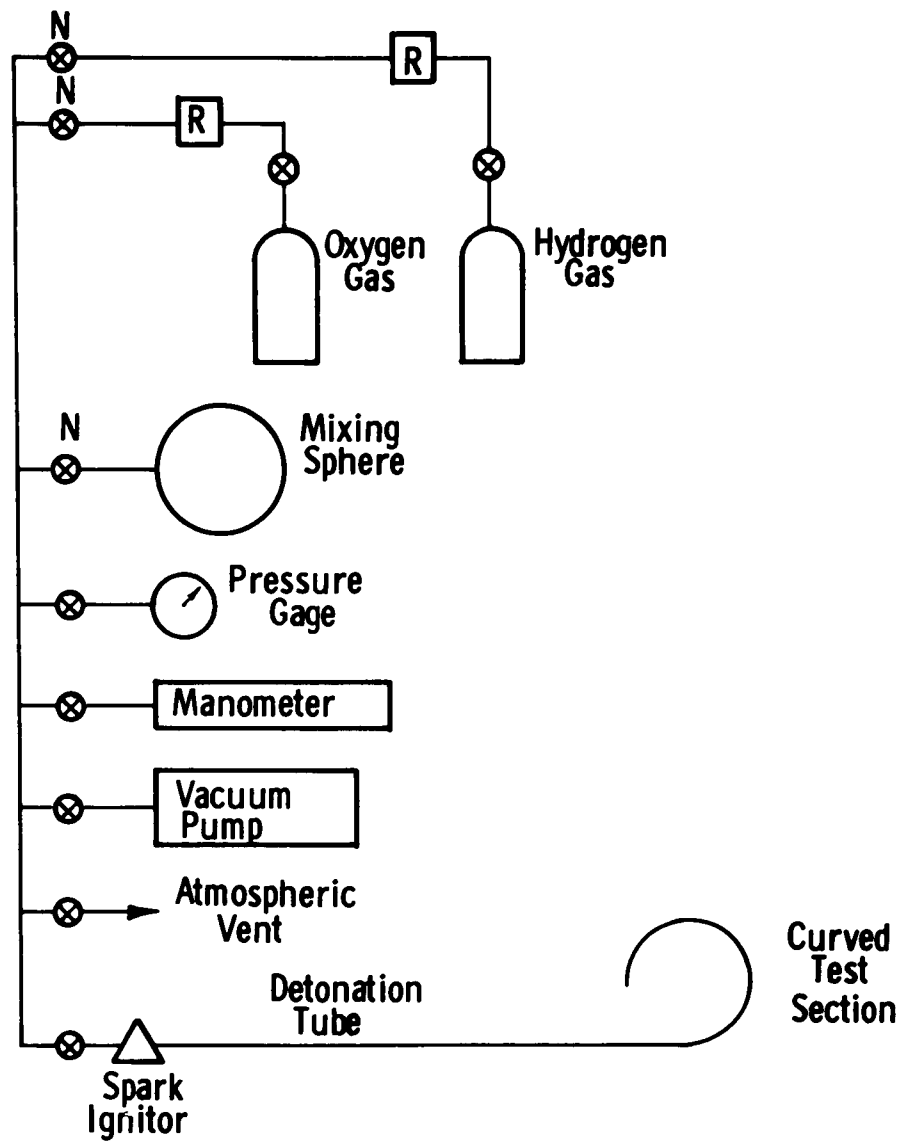


Fig. 15b. Sketch of detonation tube with curved test section and shadowgraph system.





R - Regulator  
 ⊗ - Shut-off Valves  
 N - Metering Valves

Fig. 16. Experimental setup for the curved detonation tube.